Non-linear Monte Carlo Ray Tracing for Visualizing Warped Spacetime

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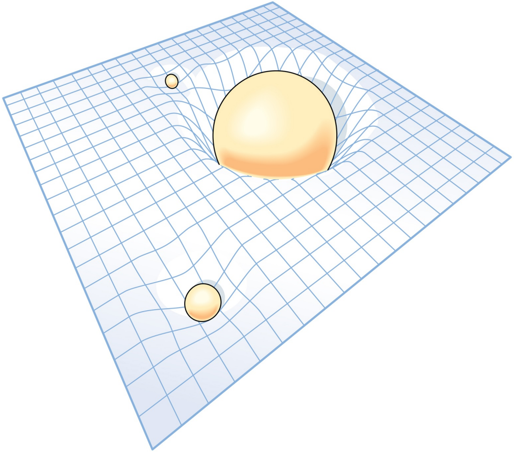
[1] Equal contribution

Fabric of Spacetime

- General relativity defines the curvature of spacetime in presence of mass and energy.
- Examples of extreme spacetime curvature include black hole and worm hole.
- Light travels along geodesics in curved spacetime.

Curved spacetime [1]



[1] Source: https://www.britannica.com/science/relativity/Curved-space-time-and-geometric-gravitation

Motivation

- How will our everyday world look like in presence of objects with strong gravitational field? Is the image on right correct?

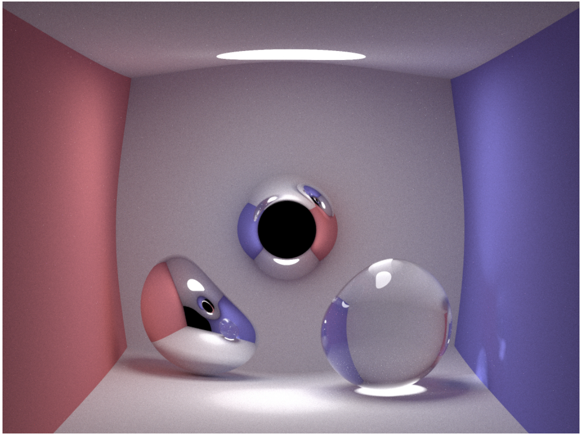
- Physically based rendering for VFX and SciViz.
- Pedagogical value - making it easier to understand curved spacetime.

Spacetime portal in Dark[1]

[1] https://www.youtube.com/watch?v=VE5BQ4Gj2jY, Netflix

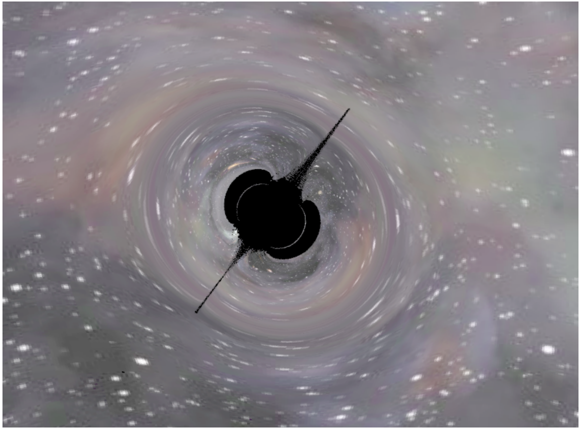
Contribution

- Non-linear ray-tracing for primary as well as secondary rays.
- Resolving all ray-object intersection with non-linear ray-tracing.
- Global illumination to render soft shadows and caustics.



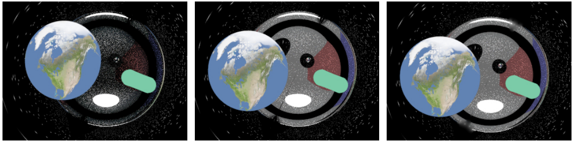
Contribution

- Our non-linear ray-tracer works at cosmic and terrestrial scales.

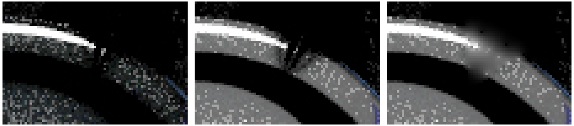


Contribution

- Bidirectional ray tracing for more efficient light transfer across for worm holes.



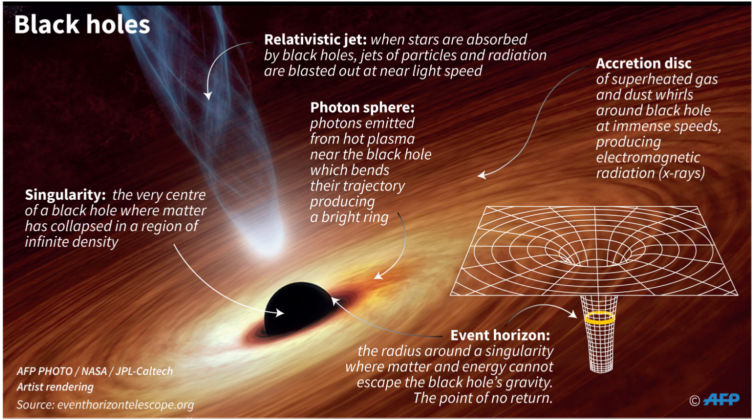
- Local smoothing to remove noise and handle singularities.



Strong Gravitational Field: Black hole

What is a Black hole?

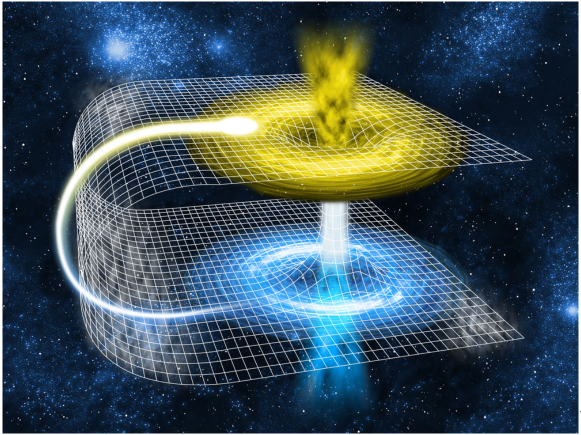
- Extremely deformed spacetime formed by highly compact mass.
- No particle or EM radiation can escape from it.
- Can be Stationary(Schwarzschild) or Rotating(Kerr).



Strong Gravitational Field: Worm hole

What is a Worm hole?

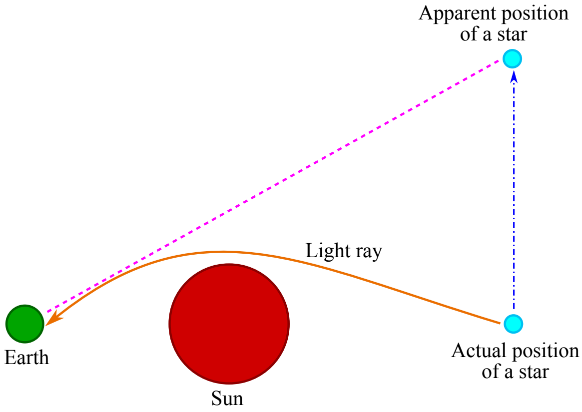
- Tunnel-like shortcuts in curved spacetime.
- Can be Traversable(Ellis) or Non-traversable(Lorentzian).


[1]

[1] (Image: © edobric | Shutterstock)

Geodesics in Curved Spacetime

- Distance between two points in a curved spacetime is given by a metric.
- Geodesics are the shortest distance between two points and are defined by the metric of that spacetime.
- Light rays follow the null geodesic.



Hamiltonian Formulation

- Hamiltonian of a spacetime manifold with metric $g^{\mu\nu}$ is

$$H(x^\alpha, p_\alpha) = \frac{1}{2} g^{\mu\nu}(x^\alpha) p_\mu p_\nu \quad \forall \alpha, \mu, \nu \in \{1, 2, 3, 4\}$$

- Equations of motion:

$$\frac{dx^\alpha}{d\zeta} = \frac{\partial H}{\partial p_\nu} = g^{\alpha\nu} p_\nu$$

$$\frac{dp_\alpha}{d\zeta} = -\frac{\partial H}{\partial x^\alpha} = -\frac{1}{2} \frac{\partial g^{\mu\nu}}{\partial x^\alpha} p_\mu p_\nu$$

Different Spacetime Metrics

- Schwarzschild metric in isotropic coordinates:

$$g_{\mu\nu} = diag\left(\left(\frac{1-\frac{r_s}{4R}}{1+\frac{r_s}{4R}}\right)^2, -\left(1+\frac{r_s}{4R}\right)^4, -\left(1+\frac{r_s}{4R}\right)^4, -\left(1+\frac{r_s}{4R}\right)^4\right)$$

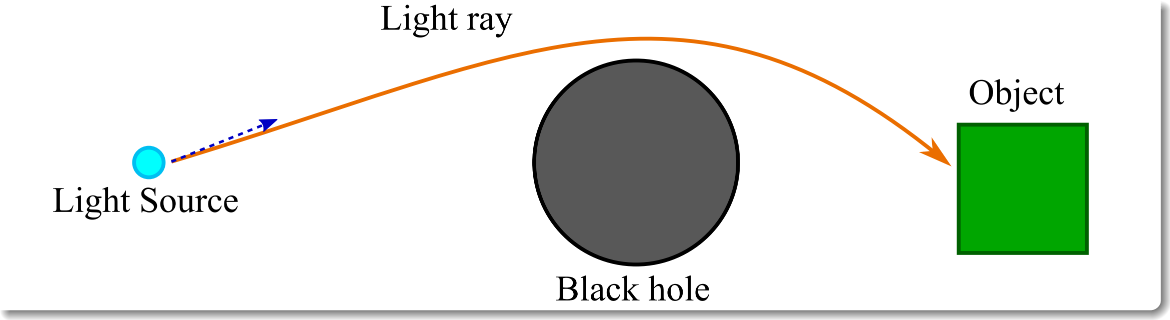
- Kerr metric in spherical coordinates:

$$g_{\mu\nu} = \begin{bmatrix} 1-\frac{2Mr}{\rho^2} & 0 & 0 & \frac{2aMr\sin^2\theta}{\rho^2} \\ 0 & -\frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ \frac{2aMr\sin^2\theta}{\rho^2} & 0 & 0 & -\left[(r^2+a^2)+\frac{2a^2Mr\sin^2\theta}{\rho^2}\right]\sin^2\theta \end{bmatrix}$$

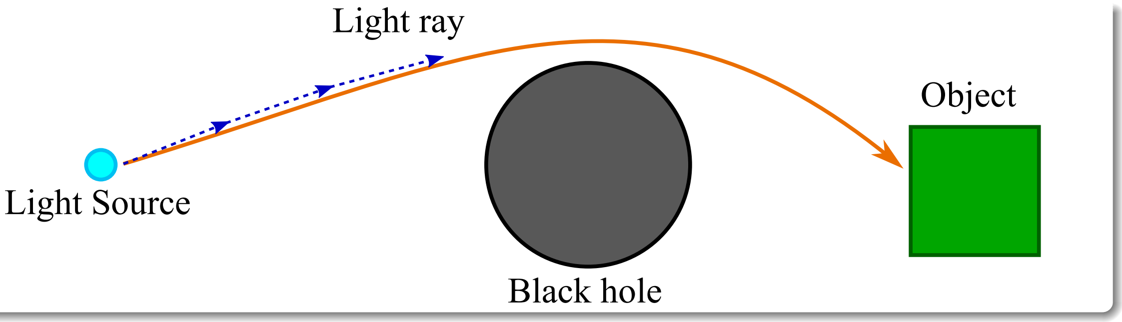
- Ellis metric in spherical coordinates:

$$g_{\mu\nu} = diag\left(1, -1, -\sqrt{\rho^2+r^2}, -\sqrt{\rho^2+r^2}\,\sin^2\theta\right)$$

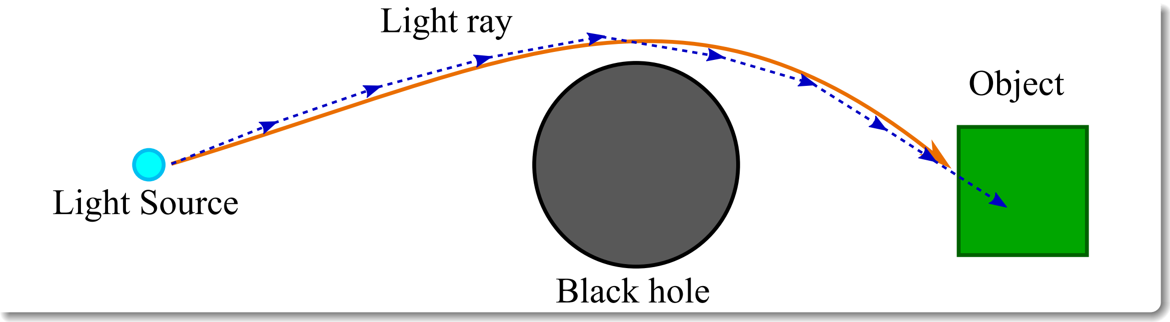
Non-linear Path Tracing Algorithm



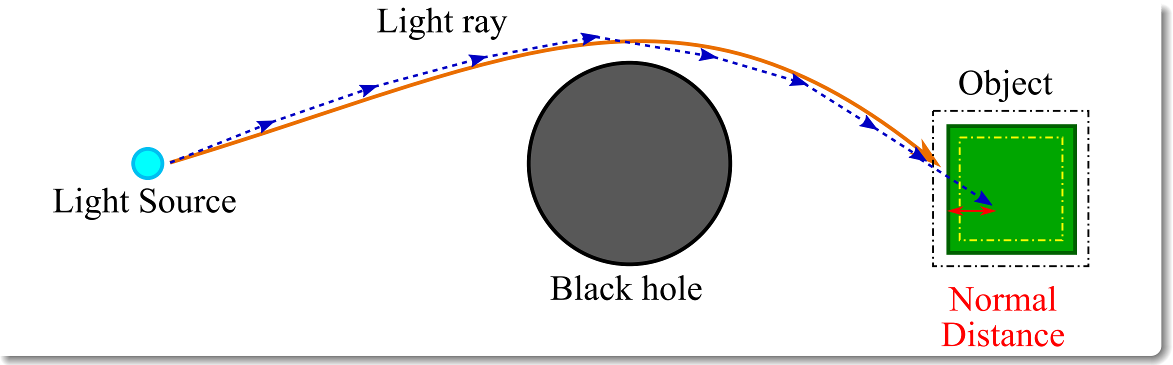
Non-linear Path Tracing Algorithm



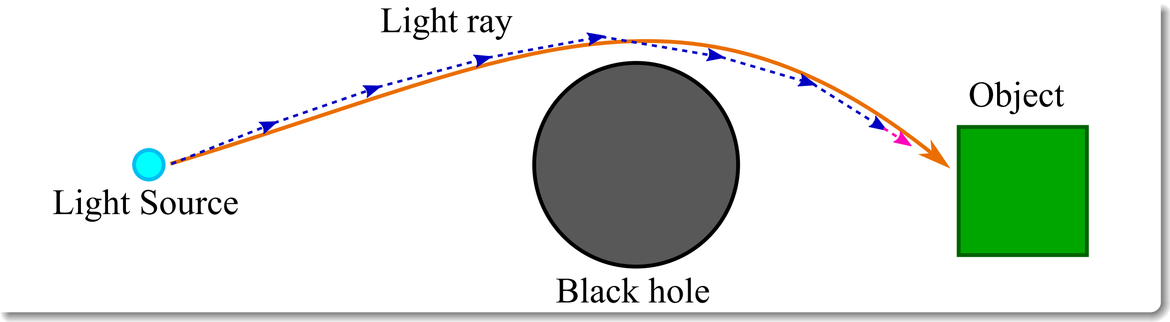
Non-linear Path Tracing Algorithm



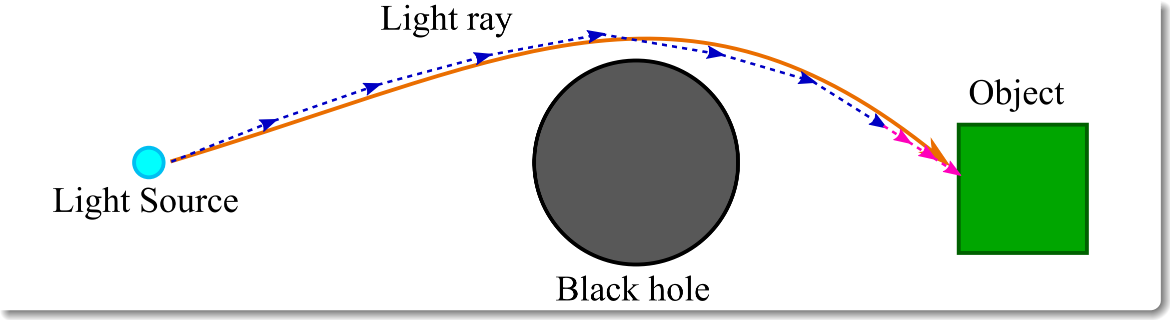
Non-linear Path Tracing Algorithm



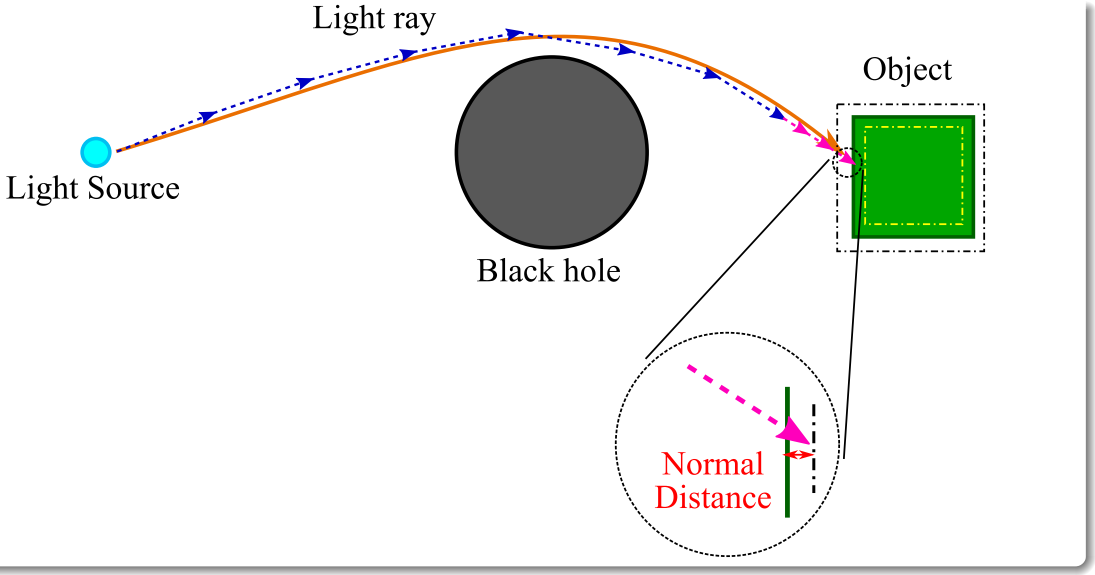
Ray-Object Intersection in Non-linear Path Tracing



Ray-Object Intersection in Non-linear Path Tracing



Ray-Object Intersection in Non-linear Path Tracing

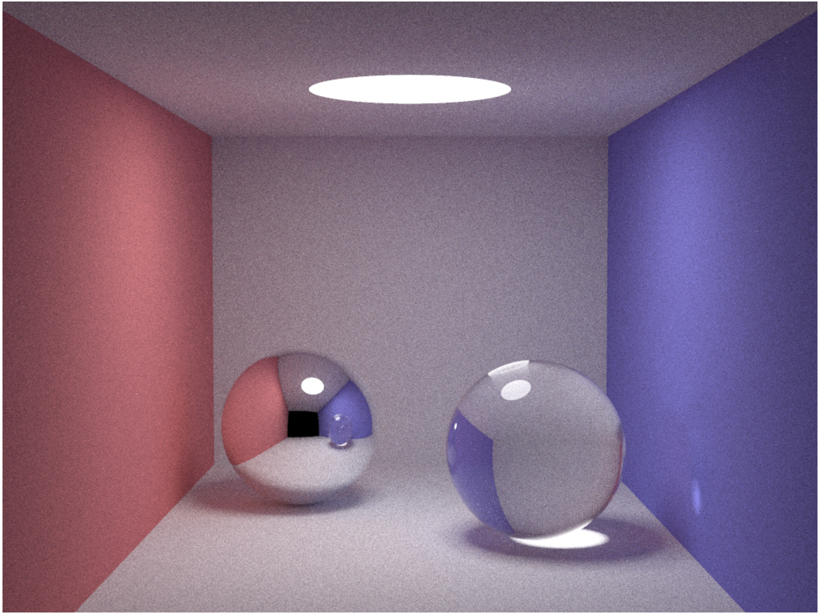


Non-linear Path Tracing Algorithm Pseudo-Code

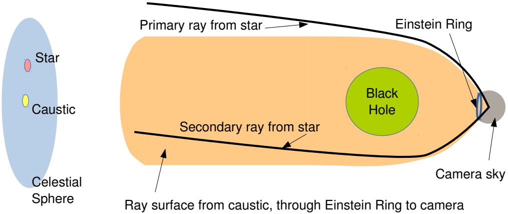
Algorithm 1 Path tracing & ray-object intersection in curved space

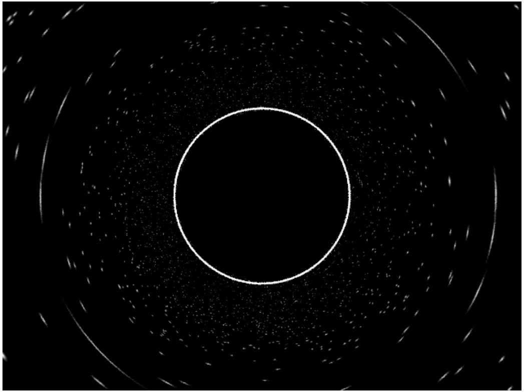
while number of iterations $<$ MAXITER **do**
    Let the current position along the Ray be x^α.
    Let current time be t and current time step be dt.
    Move forward from this position by integrating the geodesic, to a new position along the ray y^α.
    Let $\hat{n}$ be unit object surface normal.
    if $x^\alpha \cdot \hat{n} \neq y^\alpha \cdot \hat{n}$ **then**
        Find perpendicular distance, δ, to the object surface from current point.
        if $\delta < \varepsilon$ **then**
            return radiance and normal at point on surface.
        else
            Reset current position on ray to x^α.
            $dt = dt/2$.
        end if
    end if
end while
return no intersection (radiance returned is zero).

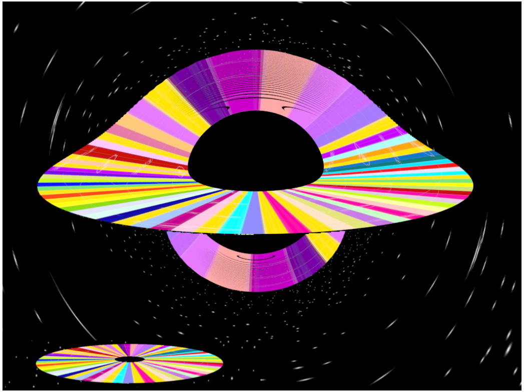
Validation Results: Flat (Minkowski) Spacetime



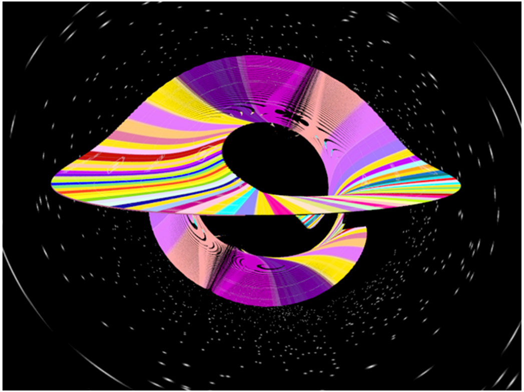
Validation Results: Einstein Ring





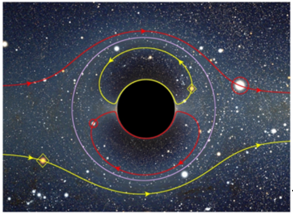


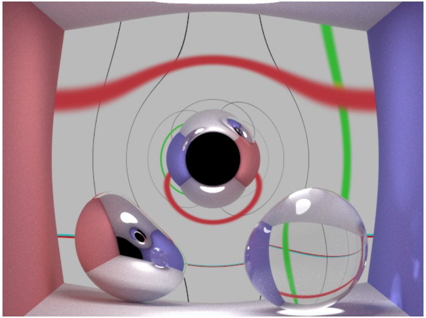
(a) Schwarzschild Black hole



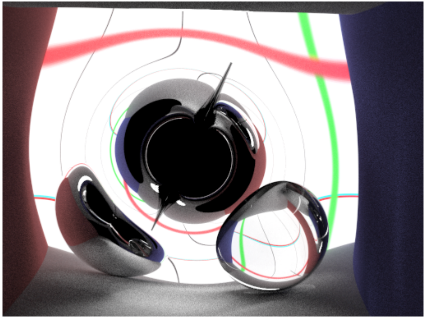
(b) Kerr Black hole

Validation Results: Primary and Secondary Images

[1]



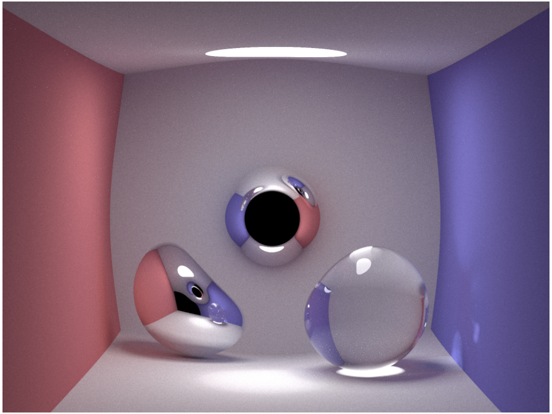
(a) Schwarzschild Black Hole

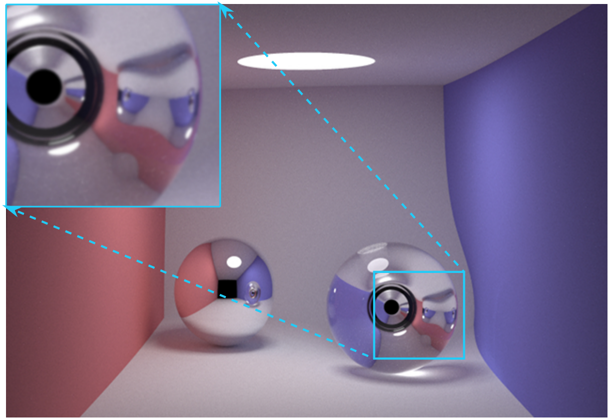


(b) Kerr Black Hole

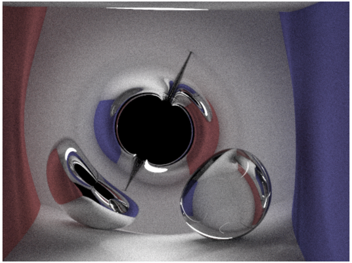
[1] https://iopscience.iop.org/article/10.1088/0264-9381/32/6/065001

Global Illumination with the Schwarzschild Black hole

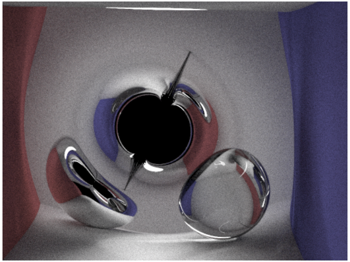




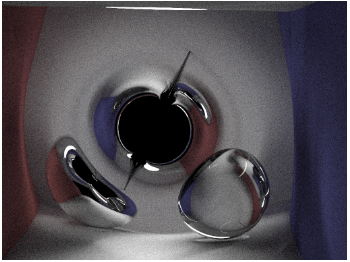
Global Illumination with the Kerr Black hole



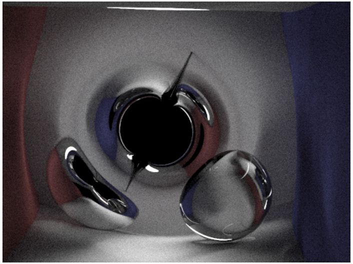
(a) a = 0.5, M = 2.5



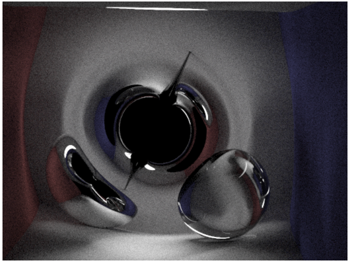
(b) a = 1.0, M = 2.5



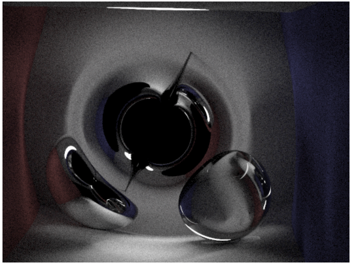
(c) a = 1.5, M = 2.5



(d) a = 1.75, M = 2.5

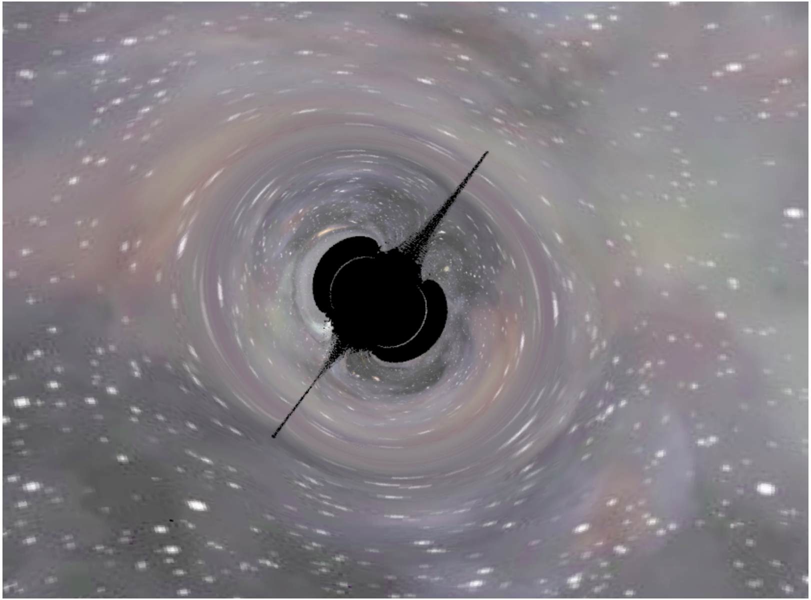


(e) a = 2.0, M = 2.5

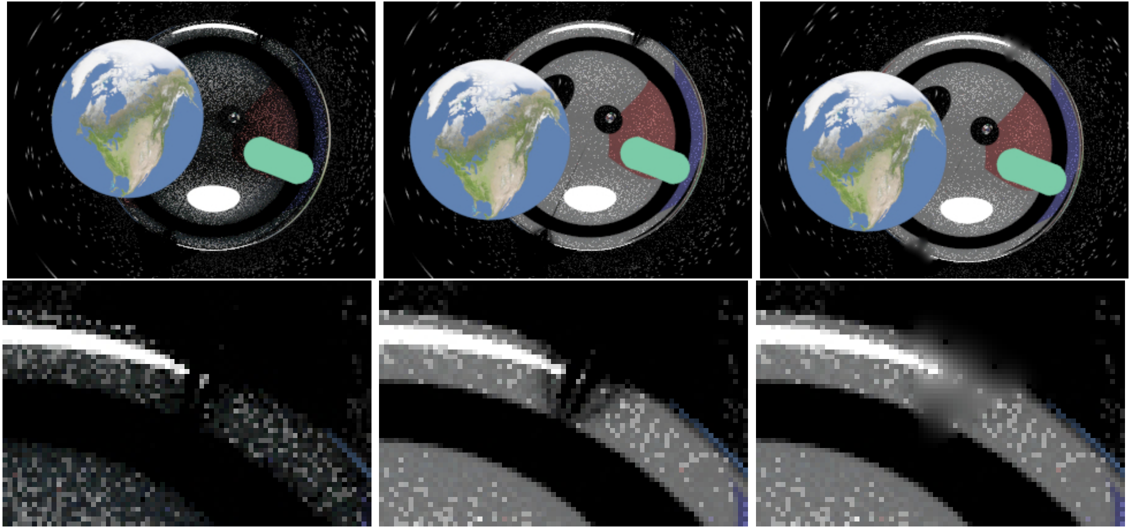


(f) a = 2.25, M = 2.5

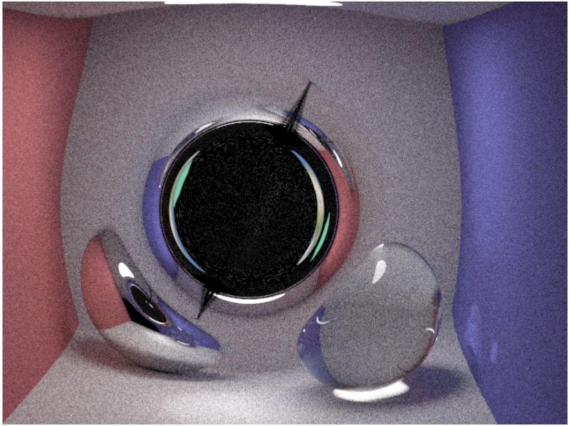
A rotating Kerr Black hole

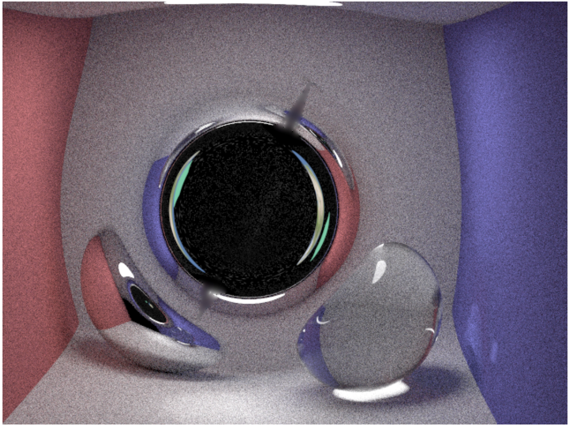


Bi-Directional Ray Tracing for the Ellis Worm hole



Local Smoothing for the Ellis Worm hole





Limitations & Future Work

- It is very time consuming to integrate along the null geodesic.
- We need more efficient sampling in curved spacetime for less noise in the images.

Thank you

Questions?